

Angles KS3 and KS4 Non-Calculator

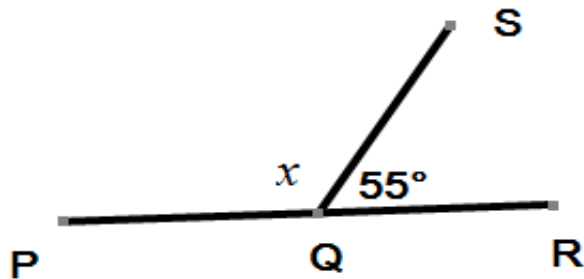
Angles in a straight line, angles at a point, vertically opposite angles, angles in a triangle, angles in a quadrilateral, angles associated with parallel lines, interior and exterior angles in polygons. * means challenging.

NOTE: All diagrams are not drawn to scale. (Do not measure angles)

You must give a reason for each answer provided.

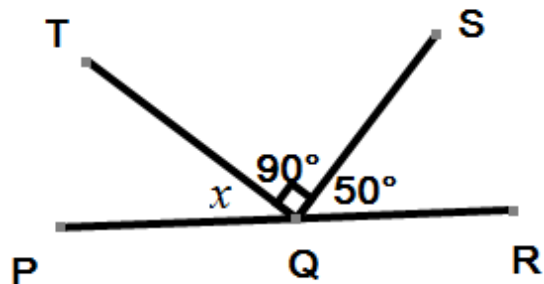
1. PQR is a straight line. Angle SQR = 55° .

Work out the size of angle marked x .



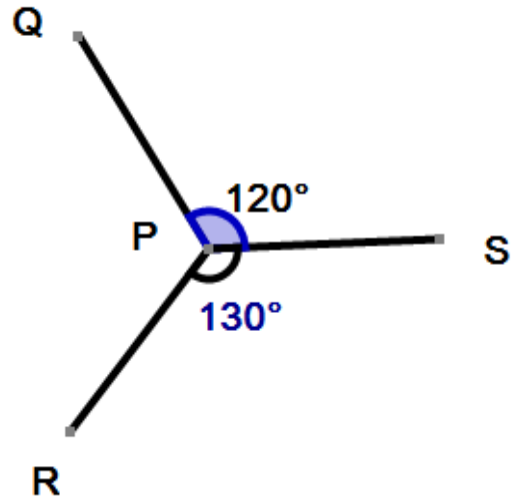
2. PQR is a straight line. Angle SQR = 50° and angle TQS = 90° .

Work out the size of angle marked x .



3. Angle QPS = 120° and angle SPR = 130° .

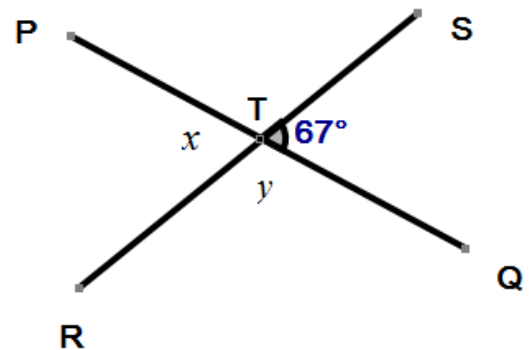
Work out the size of angle QPR.



4. The two straight lines PQ and SR meet at the point T.

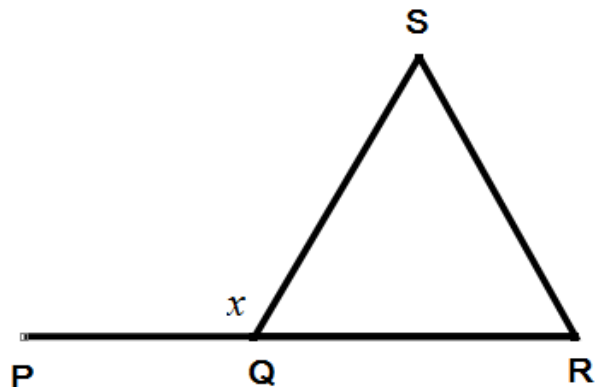
Angle STQ = 67° .

- (a) Write down the size of the angle marked x .
- (b) Work out the size of the angle marked y .



5. PQR is a straight line. Triangle SQR is an equilateral triangle.

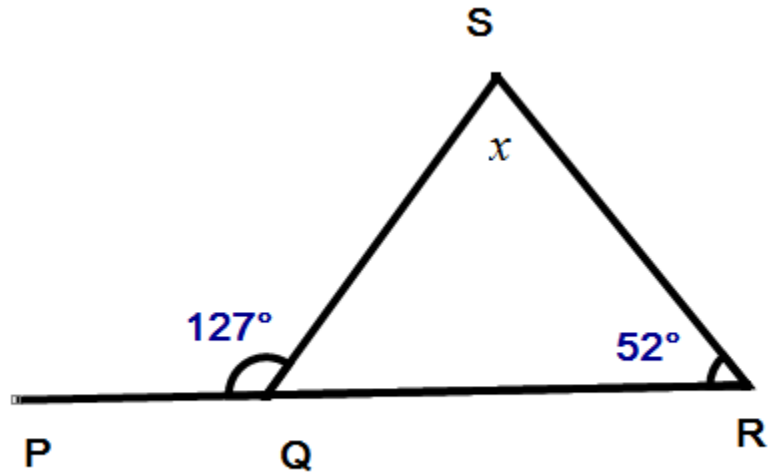
- (a) Write down the size of angle SRQ.
- (b) Work out the size of the angle marked x (PQS).



6. PQR is a straight line.

Angle QRS = 52° and angle PQR = 127° .

Work out the size of the angle marked x .



7. PQR is an isosceles triangle with $PQ = PR$.

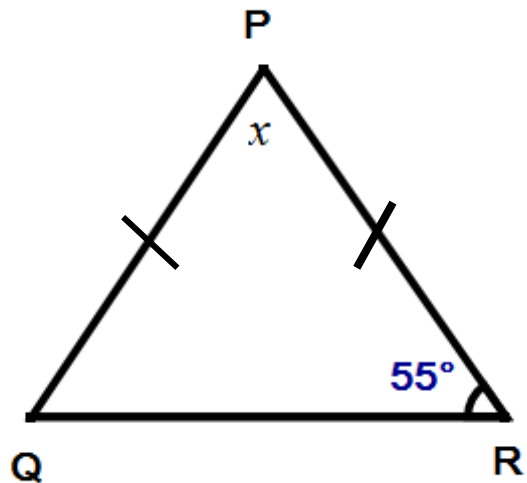
Angle PRQ = 55° .

(a) Work out the size of the angle marked x .

* (b) Given that angle PQR = y° (and not 55°),

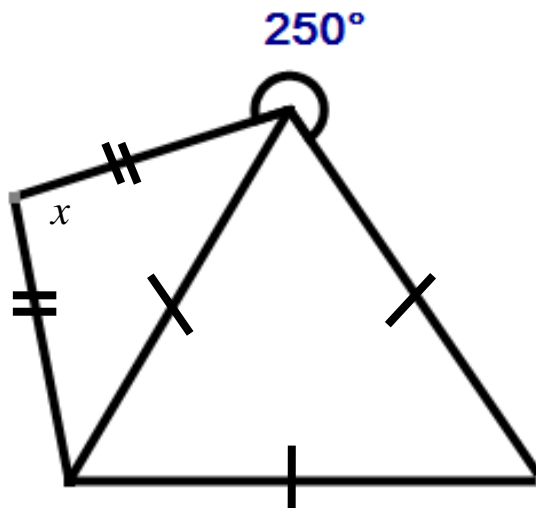
(i) Express x in terms of y

(ii) Express y in terms of x .



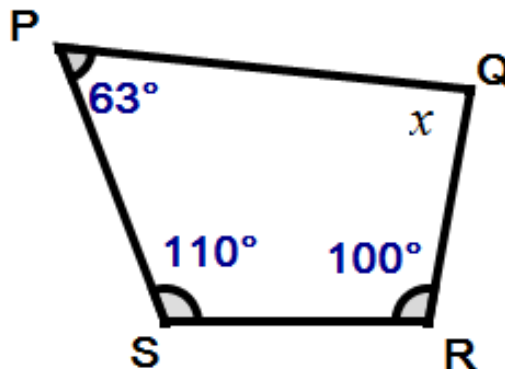
* 8. The diagram shows an equilateral triangle and an isosceles triangle.

Work out the size of the angle marked x .



9. PQRS is a quadrilateral.

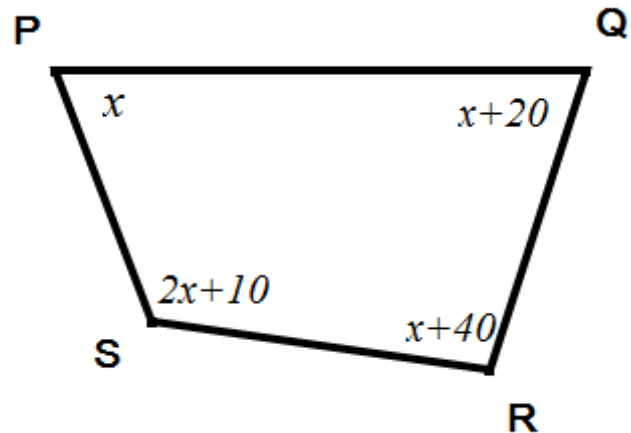
Work out the size of the angle marked x .



*10. PQRS is a quadrilateral whose angles, in degrees, are

$$x, \quad (x + 20), \quad (x + 40) \quad \text{and} \quad (2x + 10).$$

Work out the value of x .

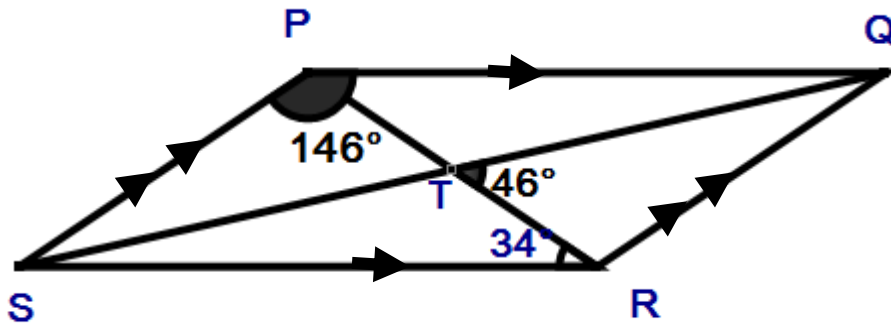


11. PQRS is a parallelogram. The diagonals PR and QS intersect at the point T.

Angle QPS = 146° , angle PRS = 34° and angle QTR = 34° .

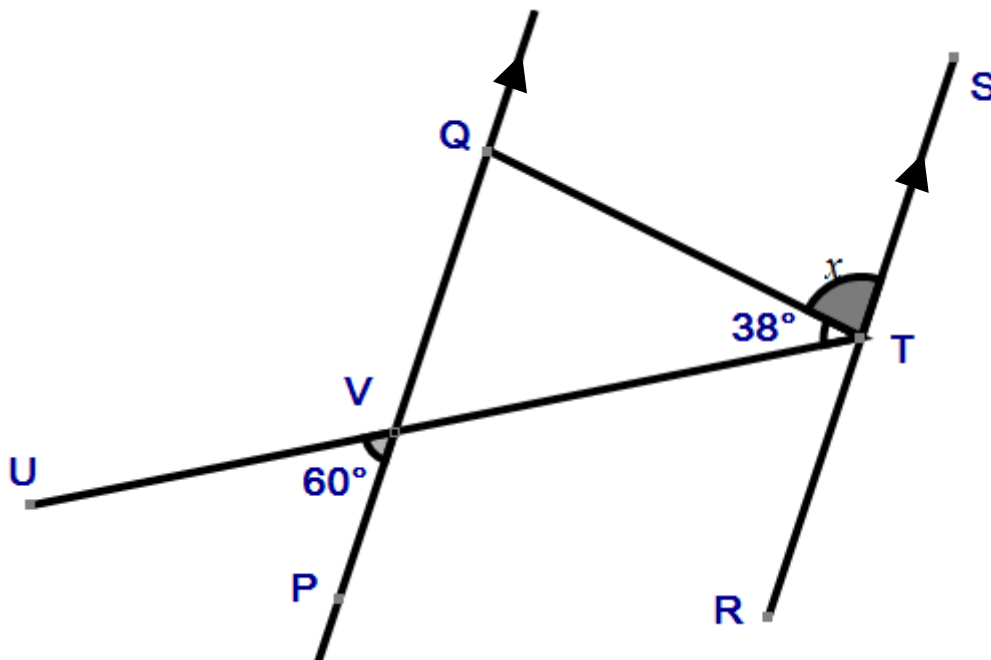
Find the size of angle:

- (a) QPT (b) SPT (c) TSR (d) QRS (e) PQR

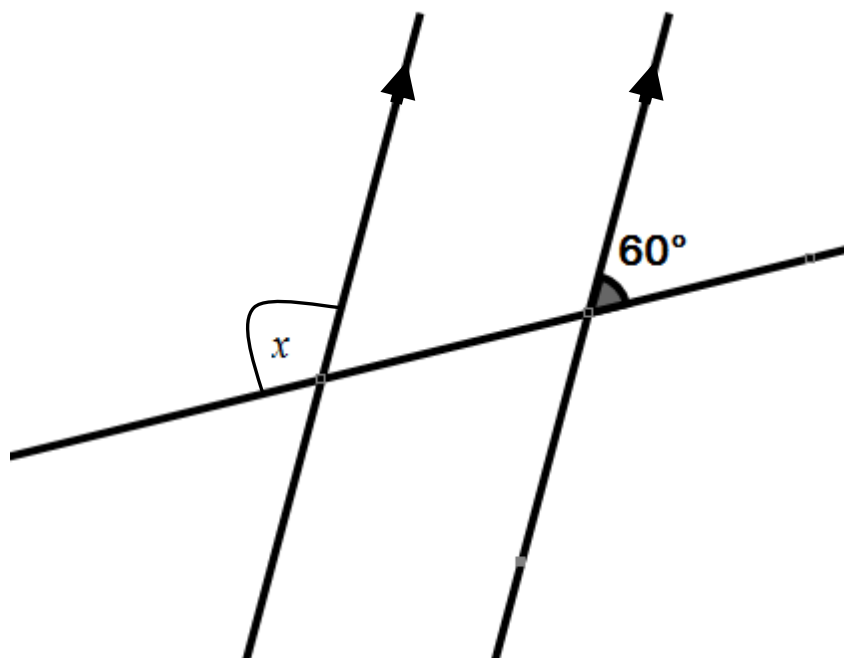


12. PQ and RS are parallel lines. Angle QTV = 38° and angle UPV = 60° .

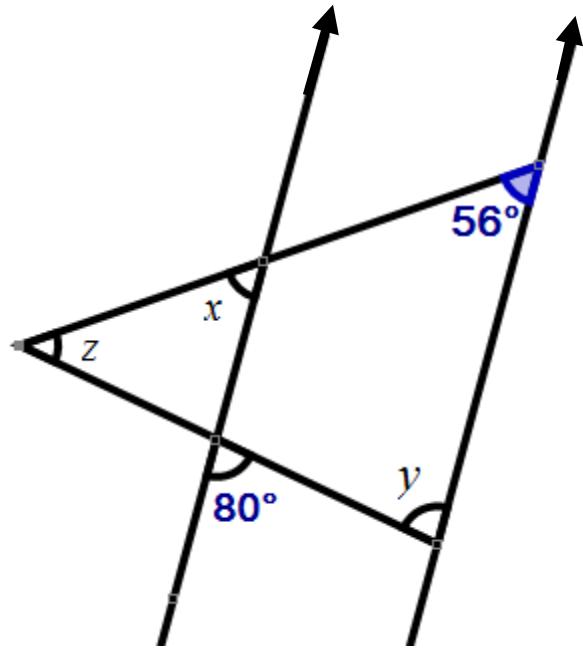
Work out the size of the angle marked x .



13. Work out the size of the marked angle x .

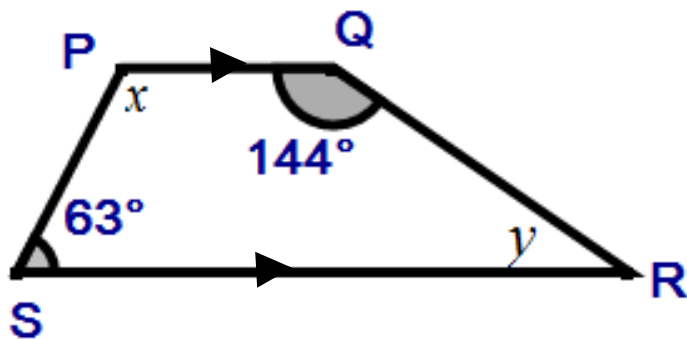


14. (a) Write down the size of each of the angles marked x and y
 (b) Work out the size of each of the angle marked z .



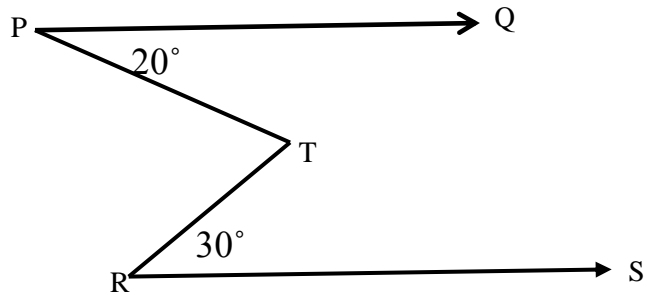
15. PQRS is a trapezium with PQ parallel to RS.

Work out the size of the angles marked x and y .



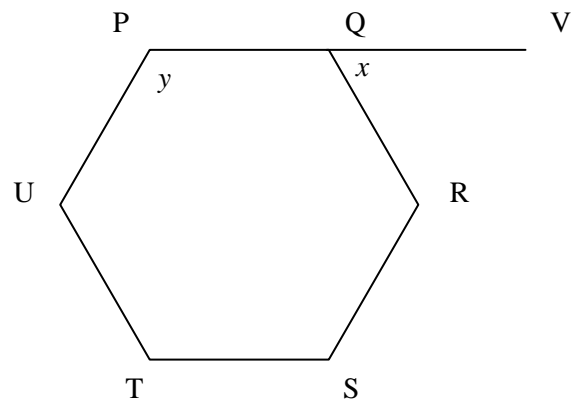
*16. PQ is parallel to RS. Angle QPT = 20° and angle TRS = 30° .

Work out the size of the reflex angle PTR.



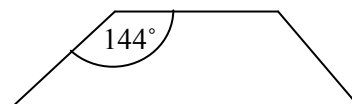
17. PQRSTU is a regular hexagon. PQV is a straight line.

- (a) Work out the size of the angles marked x and y .
- (b) Work out the size of angle PSR.
- (c) Work out the size of angle PTS.



18. The diagram shows part of a regular polygon whose interior angle is 144° .

Work out how many sides the regular polygon has.



19. The exterior angle of a regular polygon is 40° .

(a) How many sides does the regular polygon have?

(b) Work out the size of one interior angle of this regular polygon.

(c) Work out the sum of all the interior angles of this regular polygon.

20. Repeat question 19 with an exterior angle of 36° .

21. Repeat question 19 with an exterior angle of 18° .

*22. A regular polygon has 24 sides.

Work out the size of one interior angle.

*23. The interior angle of a regular polygon is 150° .

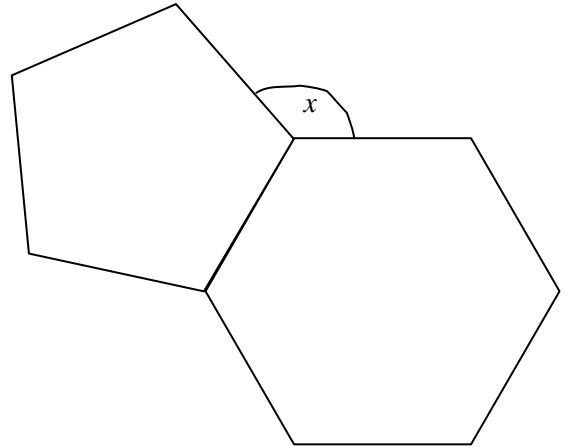
How many sides does this polygon have?

*24. The exterior angle of a regular polygon is 9° .

Work out the sum of the interior angles of this polygon.

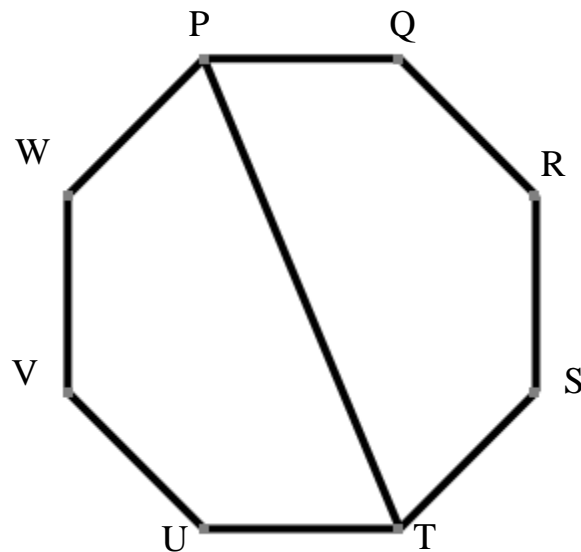
25. The diagram below shows a regular hexagon with a regular pentagon attached to one of its sides.

Work out the size of the angle marked x .

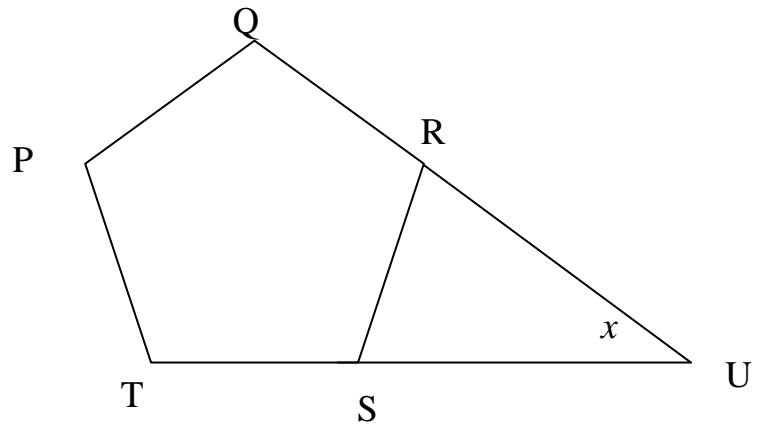


*26. PQRSTUWV is a regular octagon.

Work out the size of angle (a) PTS (b) QPR.

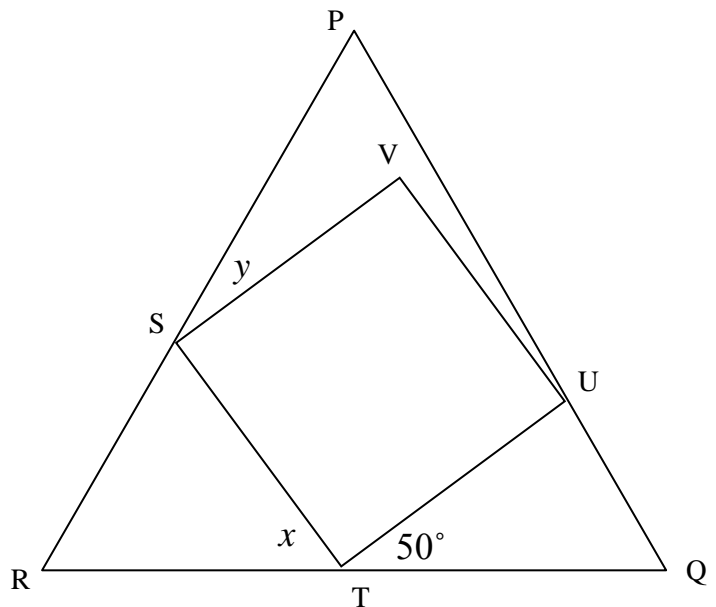


*27. PQST is a regular pentagon. QR and TS are extended (produced) to meet at the point U. Work out the size of the angle marked x .



*28. STUV is a square with three of its vertices, S, T and U on the sides of the equilateral triangle PQR.

Work out the size of the angles marked x and y .



Answers/Solutions:

Please note that not all solutions are unique.

1. $x = 180 - 55 = \mathbf{125^\circ}$. Angles in a straight line add up to 180° .

2. One angle is already, 90° . Hence, $x + 50 = 90$, $x = \mathbf{40^\circ}$.

Angles in a straight line add up to 180° . (Or $x = 180 - (90+50)$).

3. Angle QPR = $360 - (120 + 130) = 360 - 250 = \mathbf{110^\circ}$.

Angles at a point add up 360° .

4. (a) $x = \mathbf{67^\circ}$ vertically opposite angles are equal.

(b) $y = 180 - 67 = \mathbf{113^\circ}$ Angles in a straight line add up to 180° .

5. (a) Angle SRQ = $180 \div 3 = 60^\circ$. Equilateral triangle (all angles equal)

(b) Also SQR = 60° . Hence $x = 180 - 60 = \mathbf{120^\circ}$.

Angles in a straight line add up to 180° .

6. **Method 1:** Angle SQR = $180 - 127 = 53$.

Angles in a straight line add up to 180° .

Hence, $x = 180 - (53+52) = 180 - 105 = \mathbf{75^\circ}$.

Angles in a triangle add up to 180° .

Method 2: $127 = x + 52$, exterior angle property. Hence, $x = \mathbf{75^\circ}$.

7. (a) Angle PQR = 55° Isosceles triangle.

$$\text{Hence, } x = 180 - (55+55) = 180 - 110 = \mathbf{70^\circ}.$$

Angles in a triangle add up to 180° .

$$(b) \ x = 180 - 2y \quad (c) \ y = \frac{1}{2}(180 - x) \text{ or } 90 - \frac{1}{2}x$$

Isosceles triangle. Angles in a triangle add up to 180° .

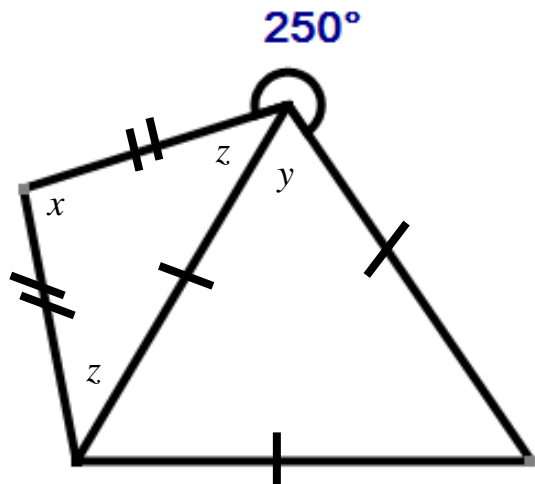
8. To make it easier, I have labeled two angles y and z on the diagram.

$$y = 60^\circ \quad \text{Equilateral triangle.}$$

$$z = 360 - (250 + 60) = 360 - 310 = 50^\circ. \text{ Angles at a point add up } 360^\circ.$$

$$\text{Hence, } x = 180 - (50 + 50) = 180 - 100 = \mathbf{80^\circ}.$$

Angles in a triangle add up to 180° .



$$9. \ x = 360 - (63 + 100 + 100)$$

$$= 360 - 263 = \mathbf{97^\circ}. \text{ Sum of angles in a quadrilateral } = 360^\circ.$$

10. $x + (x + 20) + (x + 40) + (2x + 10) = 360$ Angles in a quadrilateral.

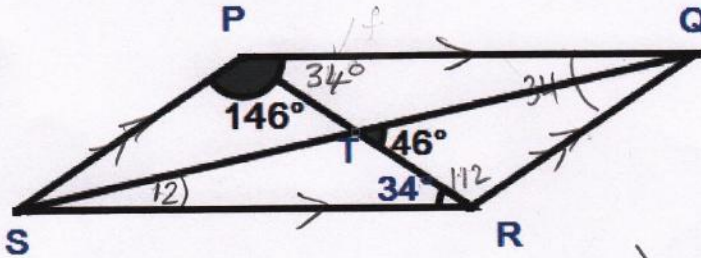
$$5x + 70 = 360$$

$$5x = 360 - 70 = 290$$

$$x = \frac{290}{5} = 58^\circ$$

11. Note

I have written the angles worked out on the diagram as it helps.



(a) $\hat{QPT} = \underline{34^\circ}$ Alternate angles (\cong angles)

(b) $\hat{SPT} = 146 - 34 = \underline{112^\circ}$ ($\hat{SPQ} = 146^\circ$ given)

(c) Method 1 $\hat{TSR} + 34^\circ = 46^\circ$ Exterior angle property
 $\hat{TSR} = \underline{12^\circ}$

Method 2
 First find $\hat{STR} = 180 - 46 = 134^\circ$ straight line (180)
 Then use $\triangle STR$: $\hat{TSR} + 34 + 134 = 180$ angles in a \triangle .
 $\hat{TSR} + 168 = 180$
 $\hat{TSR} = 180 - 168 = \underline{12^\circ}$

(d) $\hat{QRS} = \hat{PSQ} = 146^\circ$ opposite angles in a parallelogram are equal.

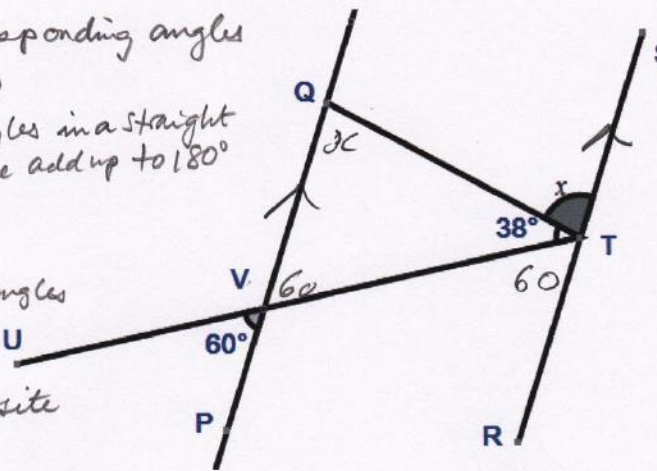
hence, using $\triangle QSR$:
 (e) $\hat{PQR} = \frac{1}{2} (360 - (146 + 146))$ angles in a quadrilateral add up to 360
 $= \frac{1}{2} (360 - 292) =$
 $= \frac{1}{2} (68) = \underline{34^\circ}$

OR $\hat{PRQ} = \hat{SPR} = 112^\circ$
 $\triangle PQR$, $\hat{PQR} = 180 - (112 + 34)$ angles in a \triangle add up to 180
 $= 180 - 146 = \underline{34^\circ}$

12. $\hat{RTV} = 60^\circ$ Corresponding angles
 $x = 180 - (60 + 38)$
 $= 180 - 98$ angles in a straight line add up to 180°
 $= \underline{\underline{82^\circ}}$

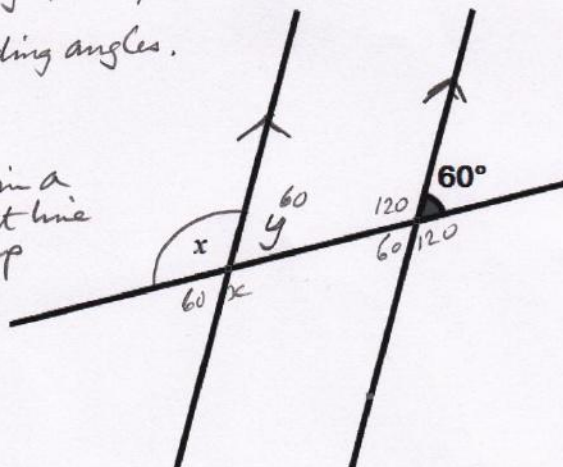
(OR)
 $\hat{VQT} = x$ alternate angles
 and
 $\hat{QVT} = 60^\circ$ vertically opposite angles

hence from ΔVTQ
 $x = 180 - (38 + 60)$ angles in a triangle add up to 180°
 $= 180 - 98$
 $= \underline{\underline{82^\circ}}$



13. I have labeled one angle y to help.
 $y = 60^\circ$ corresponding angles.

$x = 180 - 60$
 $= \underline{\underline{120^\circ}}$ angles in a straight line add up to 180

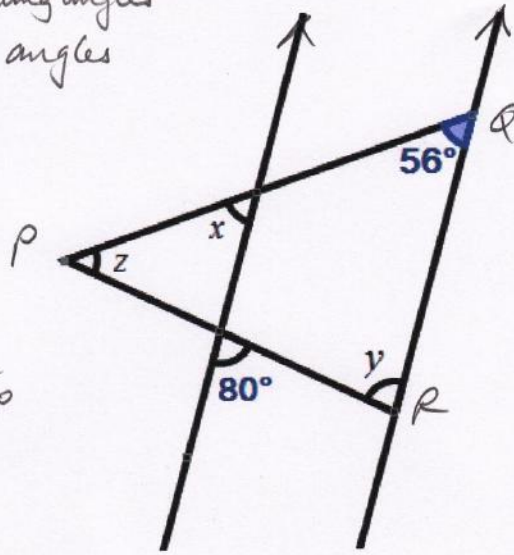


14. (a) $x = 58^\circ$ corresponding angles
 $y = 80^\circ$ alternate angles

(b) In the triangle PQR

$$\begin{aligned} z &= 180 - (56 + y) \\ &= 180 - (56 + 80) \\ &= 180 - 136 \\ &= \underline{\underline{44^\circ}} \end{aligned}$$

angles in a Δ add up to 180

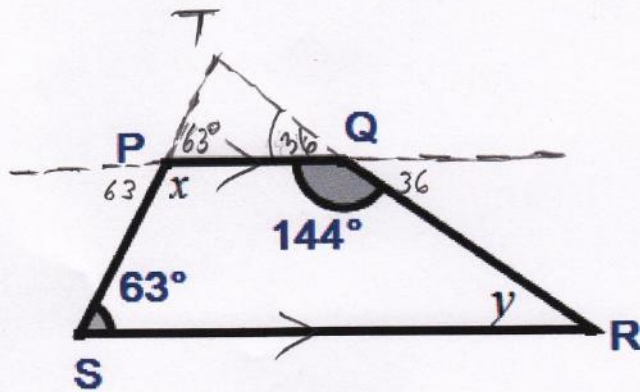


15.

$$x = 180 - 63 = \underline{\underline{117^\circ}}$$

$$y = 180 - 144 = \underline{\underline{36^\circ}}$$

Adjacent interior angles between parallel lines add up to 180 (Supplementary)



OR extend line SP and RQ to meet at T. $\hat{TPQ} = 63^\circ$ Corresponding angles

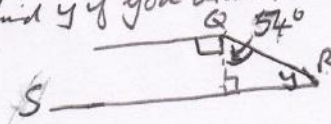
$$x = 180 - 63 = \underline{\underline{117^\circ}} \text{ straight line}$$

$$\hat{PQT} = 180 - 144 \text{ straight line} \\ = 36^\circ$$

$$y = \underline{\underline{36^\circ}} \text{ corresponding angles.}$$

I have also extended line PQ.

Can you see another way of doing this question?
 There is a 4th way to find y if you draw the height from Q onto SR



16. Draw a line through point T parallel to both PQ and RS.

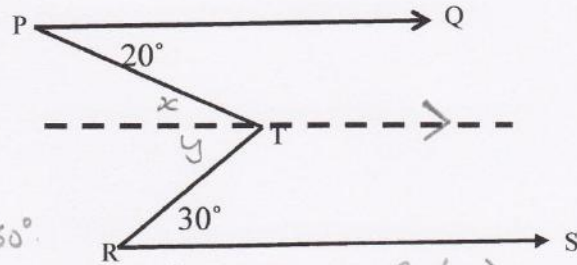
I have introduced x and y .

$$\left. \begin{array}{l} x = 20^\circ \\ y = 30^\circ \end{array} \right\} \text{alternate angles}$$

$$x + y = 50^\circ$$

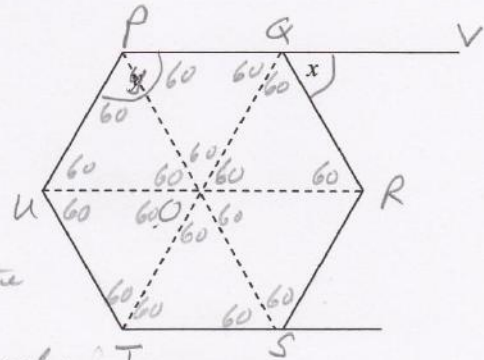
Hence reflex $\hat{PTR} = 360 - 50 = 310^\circ$

angles at a point add up to 360° .
 (See if you can find a different way of finding the angle by extending PT to meet RS.)



17. (a) exterior angle $x = 360 \div 6 = 60^\circ$
 interior angle $y = 180 - 60 = 120^\circ$
 angles in a straight line.

(b) Draw the diagonals through the centre as shown, each angle at the centre $= 360 \div 6 = 60^\circ$
 So all the triangles are equilateral. Hence $\hat{PSR} = 60^\circ$



OR $\hat{TSR} = 120^\circ$ interior angle $\hat{SR} = 120^\circ$
 PS is a line of symmetry that bisects \hat{TSR} .
 hence $\hat{PSR} = 60^\circ$

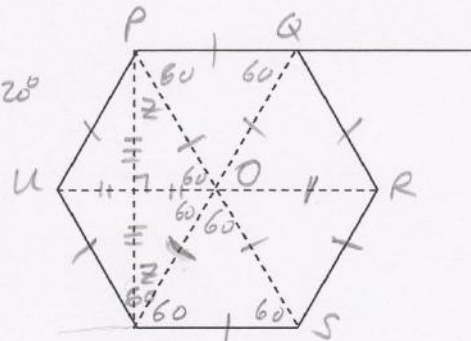
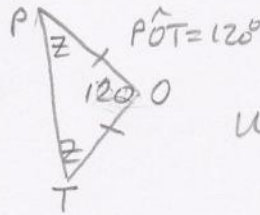
(c) Method 1 Consider $\triangle POT$!

$$2z = 180 - 120$$

$$2z = 60$$

$$z = 30$$

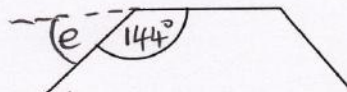
hence $\hat{PTS} = 30 + 60 = 90^\circ$



Method 2 Consider the Rhombus POTU
 The diagonals bisect each other at 90° and bisect the angles. Hence $\hat{POT} = 30^\circ$
 hence $\hat{PTS} = 30 + 60 = 90^\circ$

18. Exterior angle, $e = 180 - 144$
 $= 36^\circ$

Sum of ext. angles = 360 , $360 \div 36 = \underline{\underline{10 \text{ sides}}}$



19. (a) Sum of exterior angles = 360° .

hence there are $360 \div 40 = 9$ exterior angles, hence 9 sides

(b) interior angle = $180 - 40 = \underline{\underline{140^\circ}}$ straight line (180°)

(c) $9 \times 140 = \underline{\underline{1260^\circ}}$

20. (a) $360 \div 36 = \underline{\underline{10 \text{ sides}}}$ (same reasons as Q19)

(b) $180 - 36 = \underline{\underline{144^\circ}}$

(c) $10 \times 144 = \underline{\underline{1440^\circ}}$

21. (a) $360 \div 18 = \underline{\underline{20 \text{ sides}}}$

(b) $180 - 18 = \underline{\underline{162^\circ}}$

(c) $20 \times 162 = \underline{\underline{3240^\circ}}$

22. (a) exterior angle = $360 \div 24 = 15^\circ$
 hence interior angle = $180 - 15 = \underline{\underline{165^\circ}}$

23. exterior angle = $180 - 150 = \underline{\underline{30^\circ}}$
 number of sides = number of exterior angles
 $= 360 \div 30 = \underline{\underline{12 \text{ sides}}}$

24. number of sides = $360 \div 9 = 40$
 interior angle = $180 - 9 = \underline{\underline{171^\circ}}$
 $40 \times 171 = \underline{\underline{6840}}$

25. The diagram below shows a regular hexagon with a regular pentagon attached to one of its sides.

Work out the size of the angle marked x .

Method 1 :

$$y = \text{exterior angle of Pentagon} = 360 \div 5 = 72^\circ$$

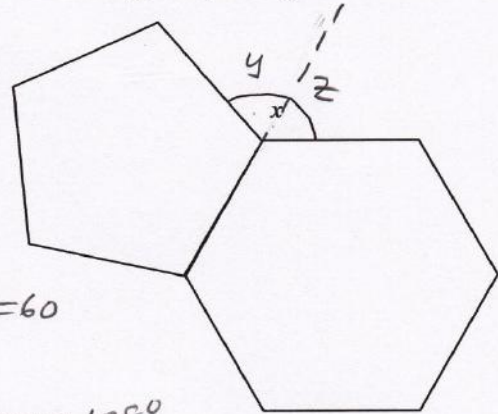
$$z = \text{exterior angle of hexagon} = 360 \div 6 = 60$$

$$x = y + z = 72 + 60 = \underline{\underline{132^\circ}}$$

Method 2 interior angle of pentagon = 108°
interior angle of hexagon = 120

$$x = 360 - 228 = \underline{\underline{132^\circ}}$$

angles at a point add up to 360°



- *26. PQRSTUWV is a regular octagon.

Work out the size of angle (a) PTS (b) QPR.

- (a) $e = \text{exterior angle of octagon}$

$$e = 360 \div 8 = \underline{\underline{45^\circ}}$$

interior angle
 $\angle TQS = 180 - 45$
 $= \underline{\underline{135^\circ}}$

By symmetry

$$\angle PTS = 135 \div 2$$

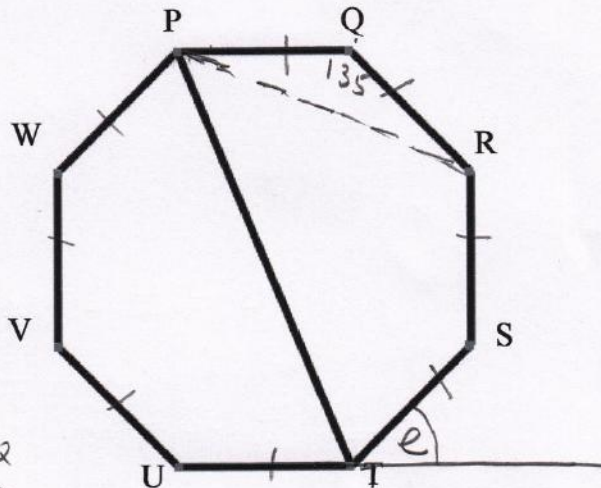
$$= \underline{\underline{67.5^\circ}}$$

- (b)

$$2x = 180 - 135$$

$$2x = 45$$

$$x = 22.5 \quad \angle QPR = \underline{\underline{22.5^\circ}}$$



27. exterior angle of
the pentagon = $360 \div 5$

$$= 72^\circ$$

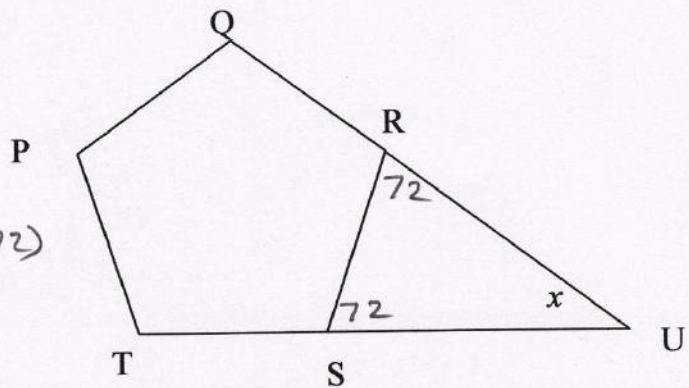
$$\hat{SRU} = 72^\circ = \hat{RSU}$$

$$\text{hence } x = 180 - (72 + 72)$$

$$= 180 - 144$$

$$x = \underline{36^\circ}$$

angles in a Δ add up to 180.



28. RTO is a straight line.
hence

$$x = 180 - (90 + 50) \quad \text{OR } 90 - 50 = 40$$

$$= 180 - 140$$

$$= \underline{40^\circ}$$

$\hat{SRT} = 60^\circ$ equilateral Δ PQR

$$\text{hence } \hat{RST} = 180 - (60 + 40)$$

$$= 180 - 100$$

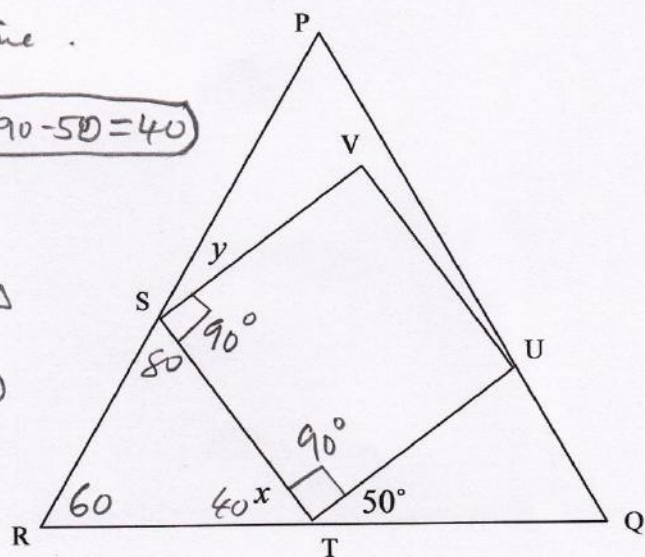
$$= 80$$

RSP is a straight line

$$\Rightarrow y = 180 - (90 + 80) \quad \text{(OR } y = 90 - 80 = 10)$$

$$y = 180 - 170^\circ$$

$$= \underline{10^\circ}$$



I hope you find this useful. If you find any errors, please let me know.

Please do leave a positive comment. It helps!!!