

38 2-D puzzles

You will revise

- ◆ the properties of special types of triangles and quadrilaterals
- ◆ angle properties around a point, on a line and with parallel lines
- ◆ the sum of the angles in a triangle
- ◆ how to draw polygons accurately from sketches

This work will help you work with

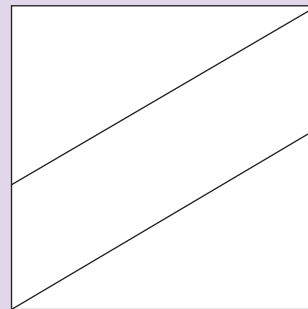
- ◆ the angles at the centre of a polygon
- ◆ the exterior angles of a polygon
- ◆ the interior angles of a polygon

A Different shapes

Squaring off

This diagram shows a square divided into two right-angled triangles and a parallelogram.

Using straight lines can you divide a square up into ...



A Two right-angled triangles

B A square and four right-angled triangles

C A kite and two congruent right-angled triangles

D Three right-angled triangles

E An isosceles triangle and two right-angled triangles

F Two trapeziums

G Two isosceles triangles and two trapeziums

H A rhombus and two arrowheads

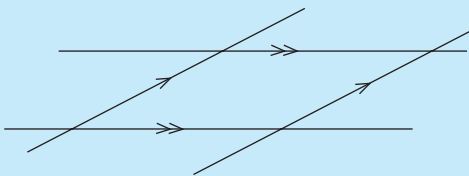
Being square

Sheet G54 has two sets of puzzle pieces.

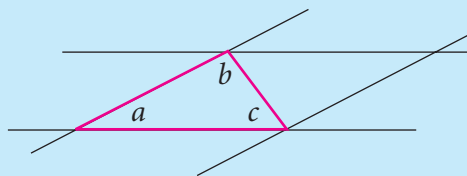
- Describe each of the shapes in the puzzles.
- Describe the symmetry of each shape and any special properties it has.

Each set of pieces can be put together to make a square.
Can you do it?

B Angles in a triangle



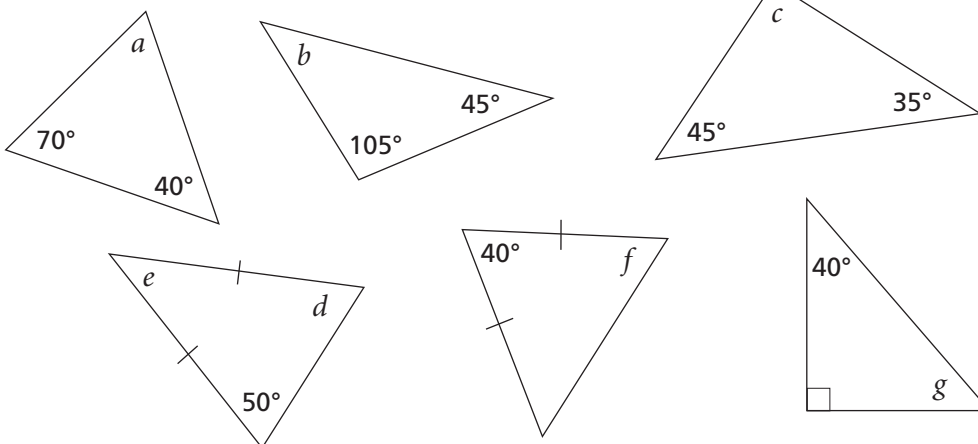
This diagram shows two sets of parallel lines which cross each other.



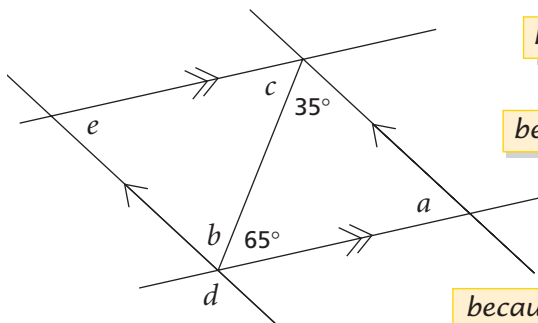
By joining two opposite corners a triangle can be drawn with angles a , b and c .

- Which other angles on the diagram are the same as a , b or c ? Explain how you know this.
- Are there any places where angles make a straight line or go round a point? What does this tell you about angles a , b and c ?

B1 Calculate the angles labelled by letters in these triangles.



B2 Find the angles labelled by letters in this diagram. Explain how you know using one of the explanations on the right.



because angles in a triangle add up to 180°

because it is an opposite angle

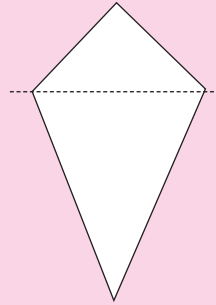
because it is an alternate angle

because angles on a straight line add up to 180°

C Angles in a quadrilateral

Here is a kite split into triangles.

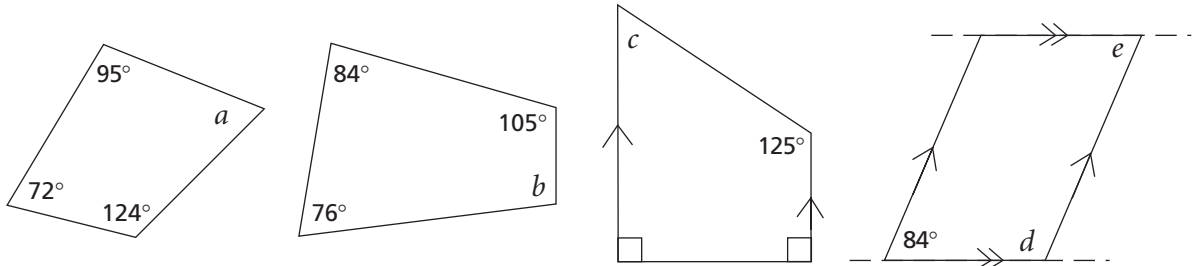
- What type are the two triangles?
- Can you split a kite into two triangles in a different way?
What types of triangle are made this time?



On sheet G55 are some other quadrilaterals.

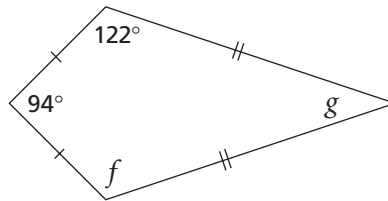
- Write the name by each one.
- Split each quadrilateral to make two triangles.
What kinds of triangle are made each time?
Are there different ways? Can any not be split into two triangles?
- If the sum of the angles in each triangle is 180° , what does this tell you about the sum of the angles in a quadrilateral?

C1 Find the angles labelled by letters in these quadrilaterals.

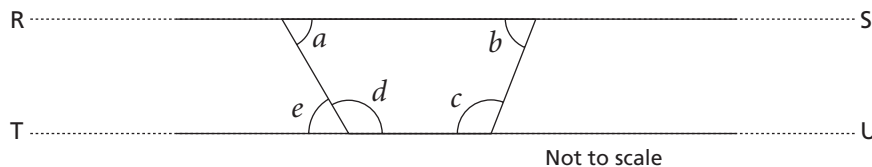


C2 This diagram shows a kite.

- What is angle f ?
Explain how you know.
- Calculate angle g .



C3 In the diagram lines RS and TU are parallel.

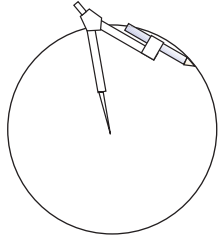


- What is the sum of the angles marked a , b , c and d ?
- Angle $a = 67^\circ$ and $c = 115^\circ$.
 - What is the size of angle e ?
 - What is the size of angle b ?

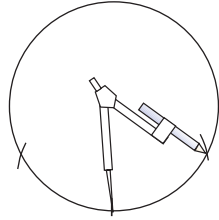
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D At the centre

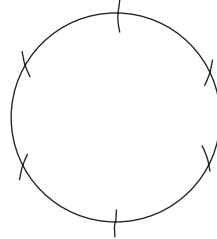
Here is a simple way to draw a regular hexagon inside a circle.



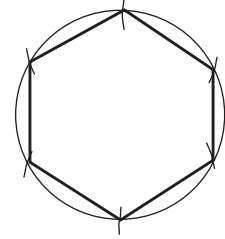
Draw a circle of radius 6 cm.



Mark a point on the circle and two spaced 6 cm each side.

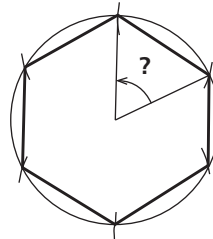


Repeat to give six equally spaced marks.

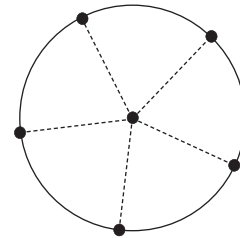


Join the marks up to make a hexagon.

Join the corners of the hexagon to the centre.
How many of these angles are there at the centre?
What must the sum of these be?
So how big are each of the angles?
Check by measuring.



- D1** To draw a regular pentagon there would need to be five points equally spaced around a circle.
- If there were lines from the points to the centre of the circle, how many angles would be at the centre?
 - What must these angles add up to?
 - Work out the size of one angle at the centre.



- D2** Work out the angles at the centre needed for the regular polygons in this table. Copy and complete the table.

| Regular polygon | Hexagon | Pentagon | Octagon | Decagon | Dodecagon |
|---------------------|---------|----------|---------|---------|-----------|
| Number of sides | 6 | 5 | 8 | 10 | 12 |
| Angle at the centre | | | | | |

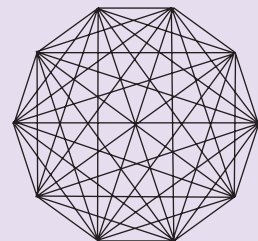
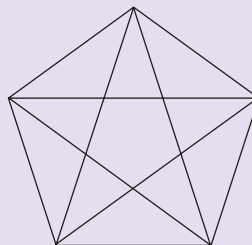
- D3** Use your table in D2 to draw each of the regular polygons inside a 6 cm radius circle. Use an angle measurer to draw the angles at the centre.

A tangled web

These patterns are made from a pentagon and a decagon.

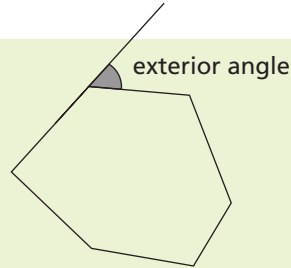
A line is drawn from each corner to every other corner.

Make some patterns of your own like these.

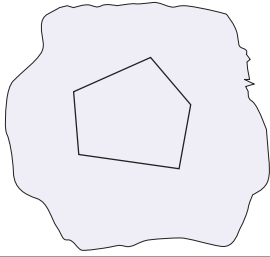


E Exterior angles

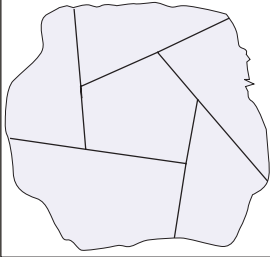
If you extend a side of a polygon an **exterior angle** is made.



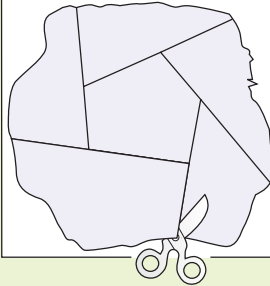
Draw a polygon in the centre of a rough circle of paper.



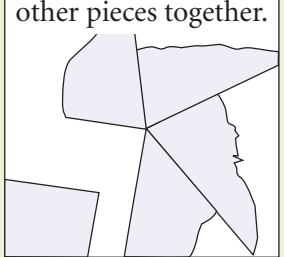
Extend each side clockwise out towards the edge of the paper.



Cut along the drawn lines.

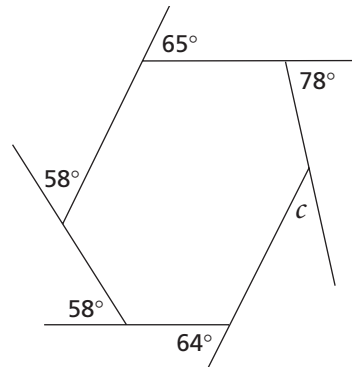
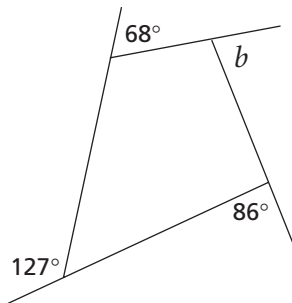
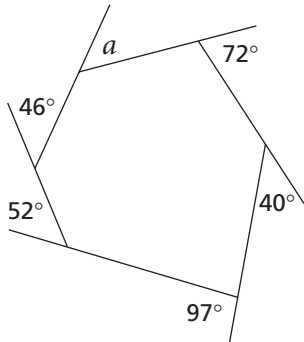


Put the polygon piece to one side. Fit the points of the other pieces together.



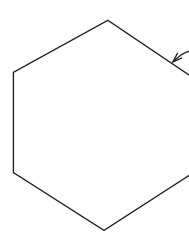
- What happens?
- Can you explain why this is true?
- Will this work for a polygon with any number of sides?

E1 Find the angles labelled by letters.



E2 This diagram shows a regular hexagon.

- How many exterior angles are there around this shape?
- What must the size of **one** exterior angle be?

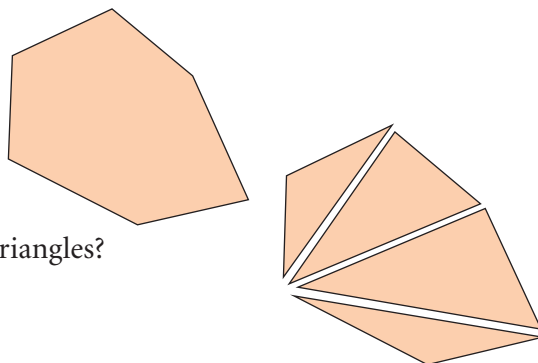


E3 Find the size of an exterior angle on

- a pentagon
- an octagon

F Interior angles

This hexagon has been split into four triangles by cutting from one corner to the other corners.



- Are there other ways the hexagon could be cut from one corner to the others to make triangles?
- What does this tell you about the sum of the angles in this hexagon?
- Draw a hexagon of your own.
How many triangles can this be split into by cutting from one corner to each other one?

- F1** (a) Draw a pentagon with a ruler – it does not have to be regular. Draw lines from a single corner to all the other corners.
- (b) How many triangles are there inside your pentagon?
- (c) What is the sum of all the interior angles of a pentagon?

- F2** (a) Draw some simple polygons to help you copy and complete the table below.

| Polygon | Hexagon | Pentagon | Octagon | Decagon | Dodecagon |
|------------------------|---------|----------|---------|---------|-----------|
| Number of triangles | | | | | |
| Sum of interior angles | | | | | |

- (b) Which of these rules describes the number of triangles that can be drawn inside a polygon if lines are drawn from one corner to all the other corners?

number of triangles = number of sides - 2

number of triangles = number of sides + 2

number of triangles = number of sides ÷ 2

number of triangles = number of sides × 2

- (c) Which of these rules describes the number of triangles (t) that can be drawn inside a polygon with n sides?

$t = n + 2$

$t = n - 2$

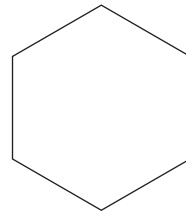
$t = 2 - n$

$t = 2n$

$t = \frac{1}{2}n$

- F3** (a) How many triangles would be made by drawing lines from one corner to every other corner in a heptagon (7 sides)?
- (b) What is the sum of all the angles in a heptagon?
- F4** (a) How many triangles would be made by drawing lines from one corner to every other corner in an icosagon (20 sides)?
- (b) What is the sum of all the angles in an icosagon?

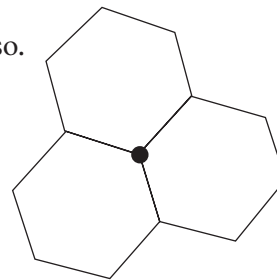
- F5** This shape is a **regular** hexagon.
This means that all the sides and angles are equal.
- (a) What is the sum of the interior angles of any hexagon?
(b) What is the interior angle of a regular hexagon?



- F6** Copy and complete this table showing the interior angle for some regular polygons.

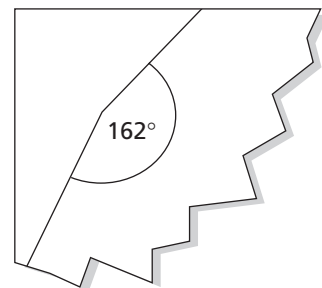
| Polygon | Hexagon | Pentagon | Octagon | Decagon | Dodecagon |
|------------------------|---------|----------|---------|---------|-----------|
| Sum of interior angles | | | | | |
| Interior of a regular | | | | | |

- F7** (a) Three regular hexagons can fit exactly round a point.
Use your answer to question F5 to explain why this is so.
- (b) Which of these combinations of shapes will fit exactly round a point?
- (i) Three regular octagons
(ii) Six equilateral triangles
(iii) Two regular octagons and a square
(iv) Four regular pentagons
(v) Two regular hexagons and two equilateral triangles



- F8** (a) The interior angle of a regular icosagon (20 sides) is 162° .
What is the exterior angle of a regular icosagon?
- (b) Use your results in sections E and F to copy and complete this table.

| Regular polygon | Interior angle | Exterior angle | Total |
|-----------------|----------------|----------------|-------|
| Hexagon | | | |
| Pentagon | | | |
| Octagon | | | |



- (c) Copy and complete this rule.

For any regular polygon the sum of the interior and exterior angle is always

G Problems and puzzles

Angles in polygons

Here are some useful facts you should have revised or learned in this unit.

Angles in a triangle

The angles in a triangle always add up to 180° .

The angles in an equilateral triangle are all 60° .

Two angles in an isosceles triangle are always the same.

Angles in a quadrilateral

The angles in a quadrilateral always add up to 360° .

Parallelograms and rhombuses have two pairs of equal angles.

A kite has one pair of equal angles.

Angles in a polygon

The **exterior** angles of any polygon always add up to 360° .

The **interior** angles of a polygon with n sides always add up to $(n - 2) \times 180^\circ$.

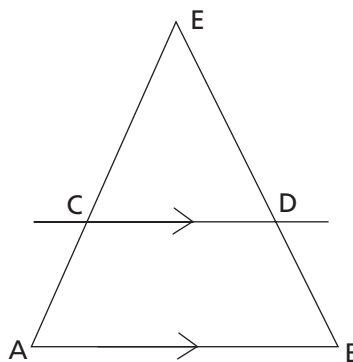
The angles from the centre of a **regular** polygon with n sides are all $360^\circ \div n$.

The exterior angles of a **regular** polygon with n sides are all $360^\circ \div n$.

The interior angles of a **regular** polygon with n sides are always $(n - 2) \times 180^\circ \div n$.

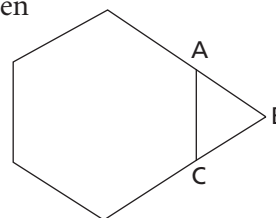
- G1** In the diagram lines AB and CD are parallel.
Length $AE = BE$.

- What special type of triangle is AEB?
- Angle AEB is 40° .
Find angle ABE.
- What type of quadrilateral is ABDC?
- Find angle BDC.
Give a reason for your answer.
- How many lines of symmetry does ABDC have?



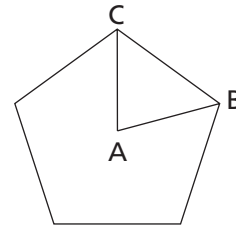
- G2** In the diagram two sides of a regular hexagon have been extended to make a triangle ABC.

- What is the interior angle of the hexagon?
- Work out angle CAB.
- What type of triangle is ABC?
Explain how you know.



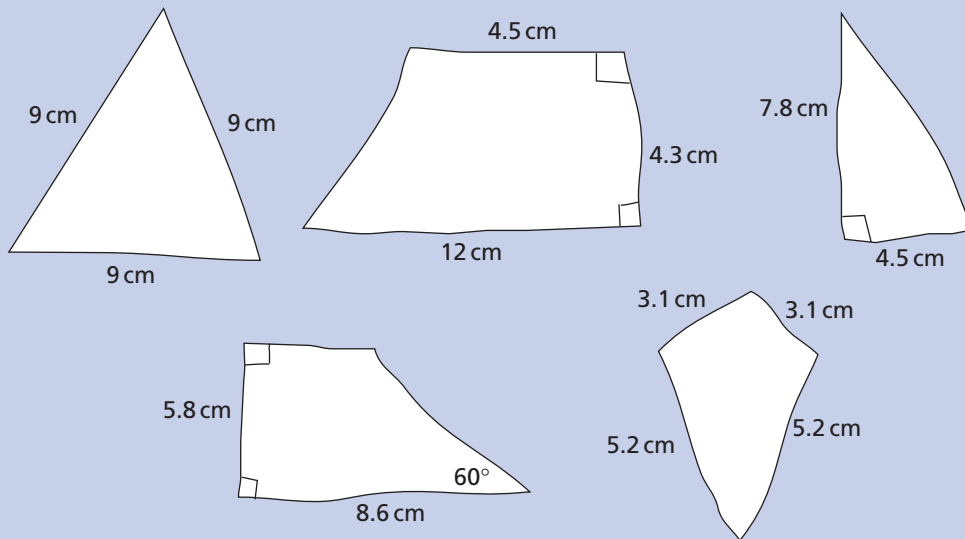
G3 In the diagram lines have been drawn from the centre of a regular pentagon to two of the corners.

- Find angle CAB at the centre of the pentagon.
- What type of triangle is ABC?
- Find angle ABC.
- Will a regular pentagon tessellate?
Give your reason.



Making squares

Make accurate copies of these shapes on plain paper.
Colour them in.



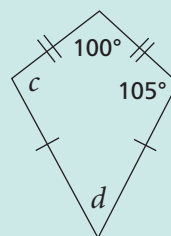
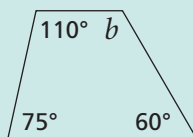
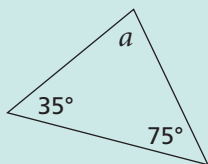
Write the name of the shape on each piece.
Can you put the pieces together to make a square?

Design your own puzzle

Use squared paper to design your own puzzle like the one in 'Making squares'.
The puzzle could be to make a square or some other shape.
Carefully transfer the design to card and cut out the shapes.
Make a sketch of each of your pieces so that someone else could make your puzzle.

Test yourself

T1 Find the angles labelled by letters in these shapes.

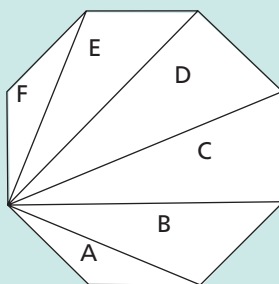


T2 Find the following.

- The sum of the interior angles in a nonagon (9 sides)
- The exterior angle of a regular nonagon
- The interior angle of a regular nonagon

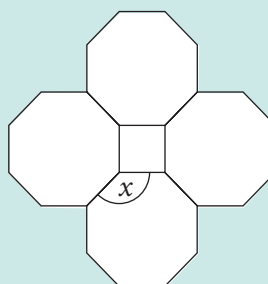
T3 Here is a regular octagon. The octagon has been divided into six triangles A, B, C, D, E and F.

- Write down a triangle that is
 - right angled
 - congruent to triangle E
 - isosceles
- In the octagon, triangles A and B make a quadrilateral.



What is the special name of this quadrilateral?

- Which **two** triangles in the octagon make a kite?
- Regular octagons and squares are used to make a tessellation.



Calculate the size of the angle marked x .

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T4 Two sides of this regular octagon have been extended to make a triangle on one of the sides.

- Find the interior angle of a regular octagon.
- Hence, or otherwise, find the **exterior** angle of a regular octagon.
- What is angle ABC?

